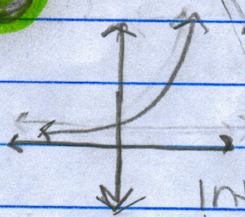
FINALS

3-1

Exponential function: $f(x) = a^x$



horozontial asymptote $y=0$

$D = \{x \mid x \in \mathbb{R}\}$

Intercept $(1,0)$

Compound interest formula: $A = P(1+\frac{r}{n})^{nt}$

A = amount P = principal n = times compounded/year

t = years r = anual interest rate (decimal)

Continous Compounding: $A = Pe^{rt}$

Natural Base = e ($e \approx 2.7182818\ldots$)

Natural exponential function: $f(x) = e^x$

3-2

Logarithmic function: $f(x) = \log_a x$

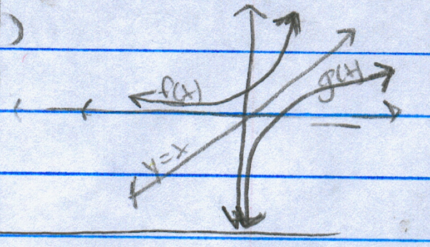
$y = \log_a x$ and $a^y = x$ are equivalent.

Common logs are expressed with base 10.

characteristic, mantissa: $\log_{10} 10 = 1.000$

Exponential Form	Log Form
$10^0 = 1$	$\log 1 = 0.000$
$10^1 = 10$	$\log 10 = 1.000$
$10^2 = 100$	$\log 100 = 2.000$
$10^3 = 1000$	$\log 1000 = 3.000$

$f(x) = a^x$ and $g(x) = \log_a x$ are inverse functions.



Properties:

1. $\log_a 1 = 0$ because $a^0 = 1$
2. $\log_a a = 1$ because $a^1 = a$
3. $\log_a a^x$ because $a^{\log_a x} = x$ (Inverse property)
4. If $\log_a x = \log_a y$, then $x = y$ (One-to-one property)

Natural Logarithmic Function: $f(x) = \log_e x = \ln x$

→ inverse function of natural exponential function

$\boxed{3\text{-}3}$

Change of Base Formula:

Base $a \to b$ Base $a \to 10$ Base $a \to e$

$\log_a x = \frac{\log_b x}{\log_b a}$ $\log_a x = \frac{\log_{10} x}{\log_{10} a}$ $\log_a x = \frac{\ln x}{\ln a}$

Properties of Logarithms:

① $\log_a(bc) = \log_a b + \log_a c$

② $\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c$

③ $\log_a b^c = c\log_a b$

$\boxed{3\text{-}4}$

One-to-one Properties

$a^x = a^y$ if and only if $x = y$

$\log_a x = \log_a y$ if and only if $x = y$

Inverse Properties

$a^{\log_a x} = x$

$\log_a a^x = x$

To solve exponential and logarithmic equations...

① Set up equations so one-to-one properties can be applied

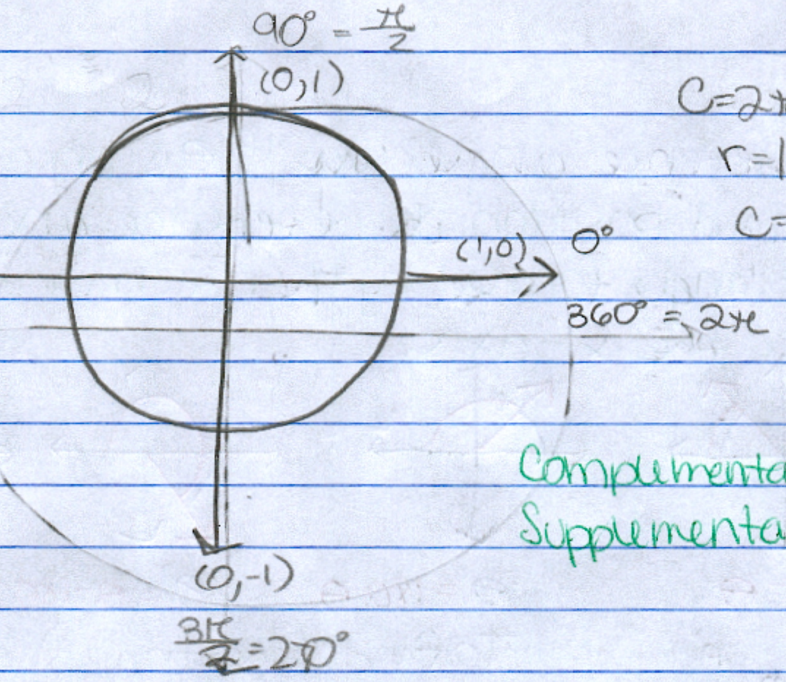
OR

② Rewrite exponential equation in log form and use inverse property

② Rewrite log equation in exponential form and use inverse property

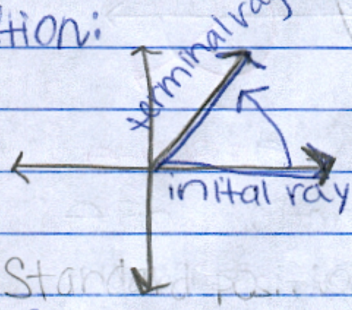
Logs of negative numbers do not exist.

4-1



$C = 2\pi r$
$r = 1$
$C = 2\pi$

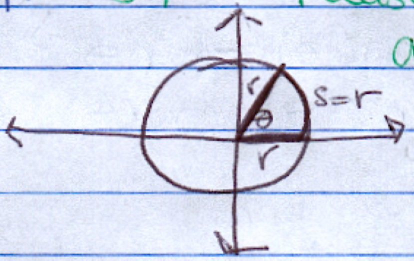
Standard Position:



Compplementary → 90°
Supplementary → 180°

Coterminal angles: Same inital + terminal sides. $\theta + 2\pi n$ / $\theta + 360 n$
(n = number of revolutions)

Radian: ~57° Measure of central angle θ when θ intercepts an arc, s, equal in length to raidus r.

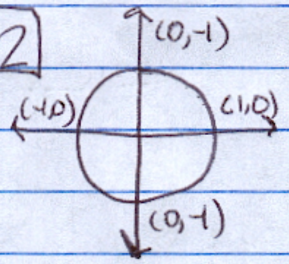


$S = 2\pi r$ ← a full revolution
arc length: $s = r\theta$

radians → degrees $\left(\frac{\pi}{180°}\right)$

degrees → radians $\left(\frac{180°}{\pi}\right)$

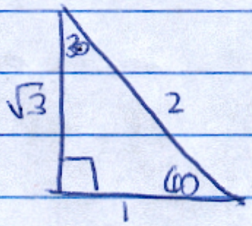
4-2

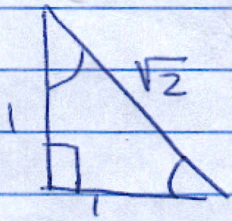


(cos, sin)
(x, y)

Periodic function: a function is periodic if $f(t+c) = f(t)$.

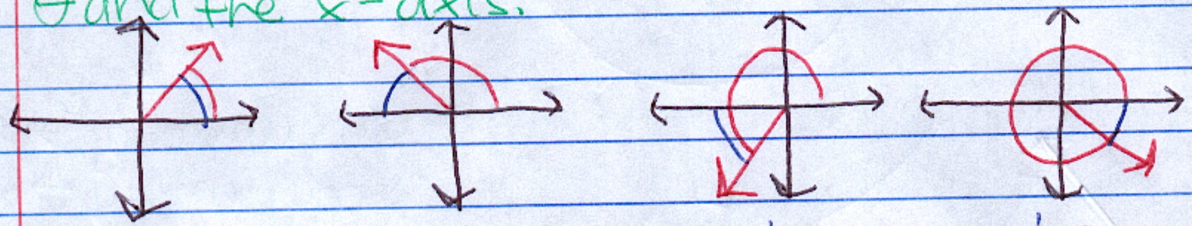
Period: The smallest number c for which t is periodic.



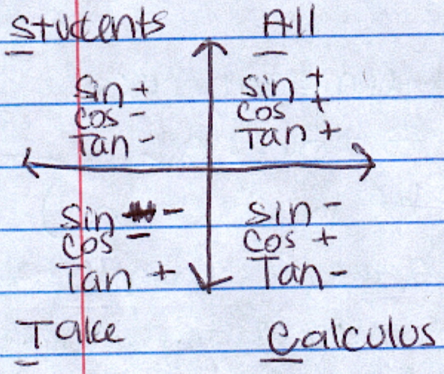


4-4

Reference angle: ~~ow~~ If θ is an angle in standard position, its reference angle is θ', an acute angle formed by the terminal side of θ and the x-axis.



$\theta' = \theta$ $\qquad$ $\theta' = 180 - \theta$ $\qquad$ $\theta' = \theta - 180$ $\qquad$ $\theta' = 360 - \theta$



To find the trig functions of θ:

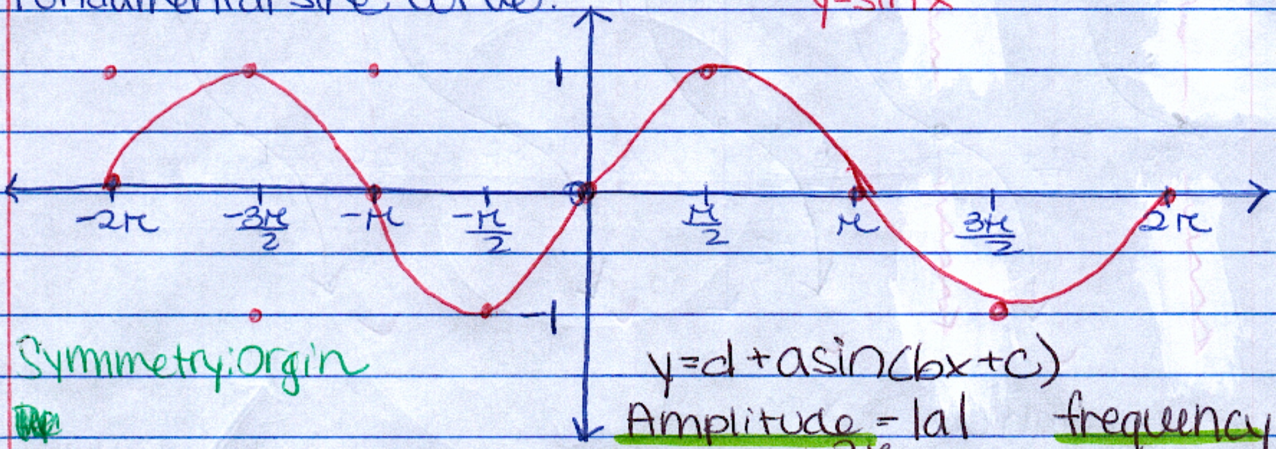
① Determine θ' and its trig functions

② Determine the sign of θ and put it on the functions of θ'.

$\sin(-\theta) = -\sin\theta$
$\tan(-\theta) = -\tan\theta$ } odd

$\cos(-\theta) = \cos(\theta)$ } eveen

Fundamental Sine Curve: $y=\sin x$



Symmetry: orgin

$y=d+a\sin(bx+c)$

Amplitude $=|a|$ frequency $=b$

Period $=\frac{2\pi}{b}$

Scale $=\Delta x=\frac{\text{period}}{4}$

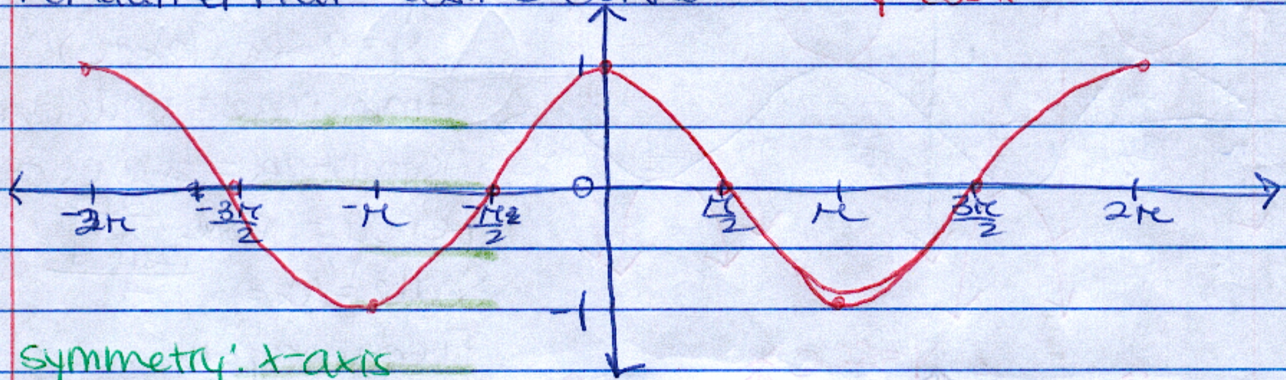
Left + right endpoints: (Phase shift) Negative → left, positive → right

$bx+c=0$ and $bx-c=2\pi$

~~Horizontal Translation:~~

Vertical Translation: d units upwards or downwards

Fundamental Cosine curve: $y=\cos x$

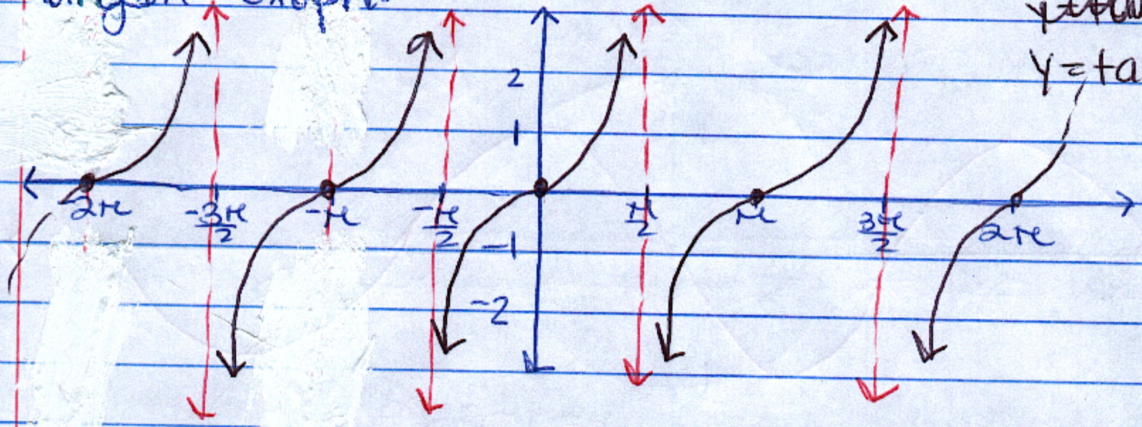


Symmetry: x-axis

Graphs osscilate over $y=d$

Tangent Graph:

~~y=tan x~~
$y = \tan x$



$y = a\tan(bx+c)$

Amplitude = undefined

Period = $\frac{\pi}{|b|}$

Scale = $\Delta x = \frac{\text{period}}{4}$

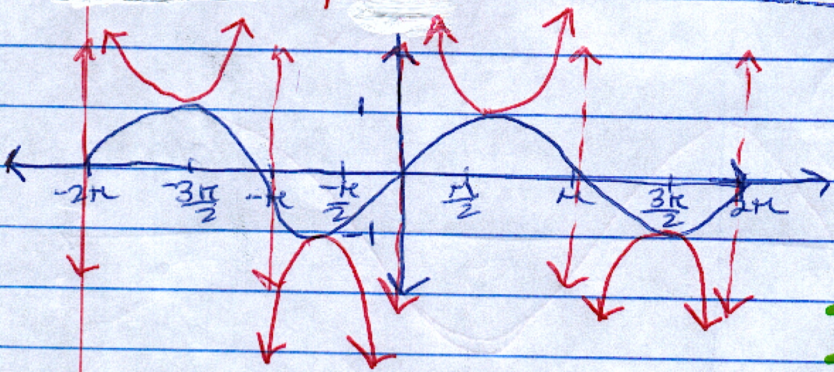
Frequency = b

Asymptotes = $bx+c = -\frac{\pi}{2}$

$bx+c = \frac{\pi}{2}$

Phase Shift = $-\frac{c}{b}$

$y = \csc x$



$y = a\csc(bx+c)$

$y = a\sec(bx+c)$

Amplitude = undefined

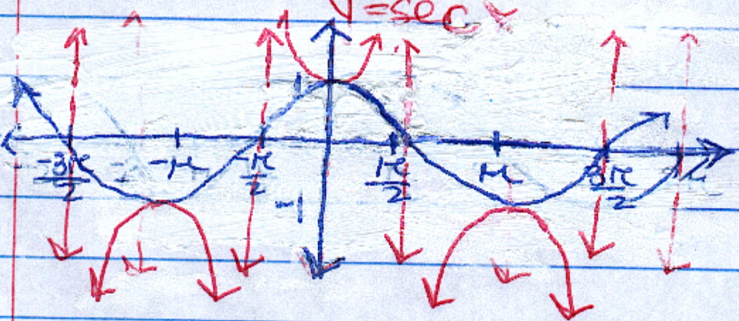
Asymptotes = zeroes of sin/cos

Period = $\frac{2\pi}{b}$

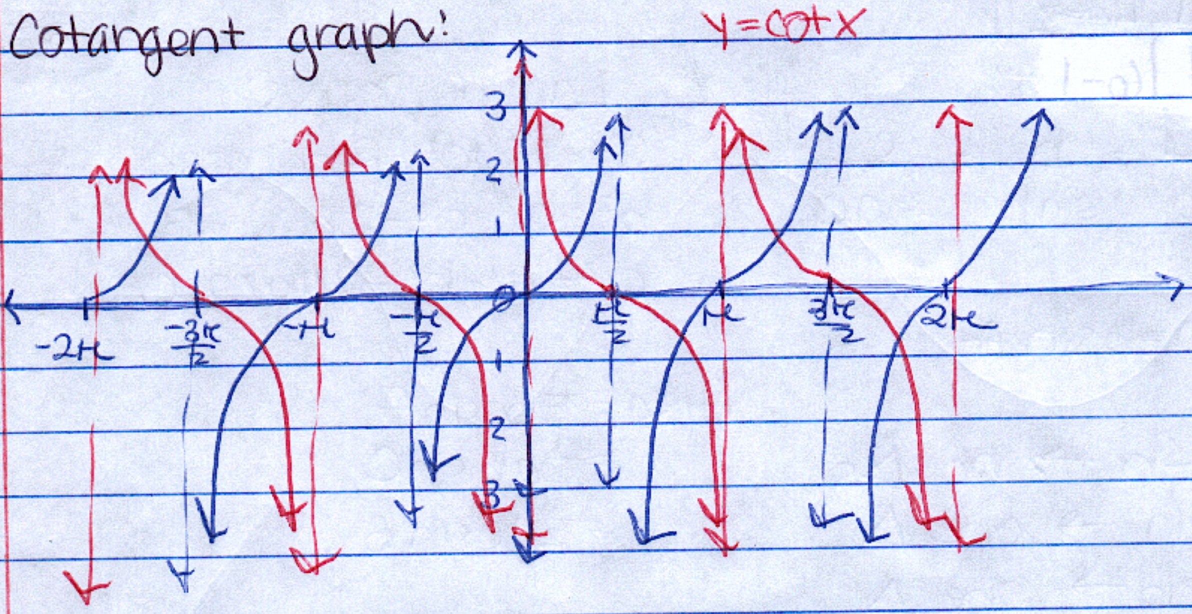
Scale = $\Delta x = \frac{\text{period}}{4}$

Frequency = b

$y = \sec x$



Cotangent graph:



$y = a\cot(bx+c)$

Asymptotes: $bx+c=0$

$bx+c=\pi$

Period $= \frac{\pi}{|b|}$

$\Delta x = \frac{\text{period}}{4}$

Phase shift $= -\frac{c}{b}$

Frequency $= b$

Amplitude = undefined

$\boxed{4\text{-}7}$

$\sin^{-1}x$ restrictions: domain $=[-1,1]$ range $=[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\cos^{-1}x$ restrictions: domain $=[-1,1]$ range $=[0,\pi]$

$\tan^{-1}x$ restrictions: domain $=(-\infty,\infty)$ range $=(-\frac{\pi}{2}, \frac{\pi}{2})$

Solving trig equations:

1. solve ~~for x~~
2. find all angles x is equal to
3. define as $\{a^\circ + b^\circ n\}$
4. let ~~n=0~~, n=0, 1, 2 → ° measure = 360°

$\boxed{6\text{-}1}$ $\boxed{6\text{-}2}$ 2ÖD

Law of sines:

$$\frac{a}{\sin a} = \frac{b}{\sin b} = \frac{c}{\sin c}$$

Law of cosines:

$a^2 = b^2 + c^2 - 2bc \cos a$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

$h = b \sin A$

Area $= \frac{1}{2} bc \sin A$

$= \frac{1}{2} ab \sin C$

$= \frac{1}{2} ac \sin B$

$a < h \rightarrow$ no Δs

$a = h \rightarrow$ one Δ

$a > b \rightarrow$ one Δ

$h < a < b \rightarrow$ Two Δs

$a \leq b \rightarrow$ no Δs

Heron's formula:

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{a+b+c}{2}$

~~Heron's~~ $\boxed{7\text{-}1}$ $\boxed{7\text{-}2}$

Solve a system by substitution:

① solve one equation for one variable

② sub into other equation to obtain one variable

③ sub back into first equation to obtain other variable

Solve a system by elimination:

① Eliminate one variable

② solve for other variable

③ Substitute back + solve for first variable

$\boxed{7\text{-}3}$ Calculator:

① Matrix (2nd, x-1)
Edit
1
Quit (2nd mode)
(Values)

② Matrix
Edit
2
Quit
(Solutions)

③ Matrix
1
~~[illegible]~~ ?
•
Matrix
2
Enter

9-1, 9-2, 9-3

~~Sequences~~

Sequence: a function with the domain of all natural numbers. The values of this function are its terms. There is a one-to-one correspondance between the natural numbers and the terms of a sequence.

Arithmetic sequence: difference between consecutive terms ~~are~~ is d, the common difference.

$a_2 - a_1 = d$ It is added to each term to get the proceeding one.

Recursive formula (using one term to generate others) $\boxed{a_{n+1} = a_n + d}$

Explicit formula:

1st term $= a_1$

2nd $= a_1 + d$

3rd $= a_1 + 2d$

4th $= a_1 + 3d$

~~n~~ nth $= \boxed{a_1 + (n-1)d = a_n}$

Geometric Sequence: difference between consecutive terms is r, the common ratio.

$\frac{a_2}{a_1} = r$ Each term is ~~multip~~ multiplied by r to get the proceeding one.

Recursive formula: $\boxed{a_{n+1} = a_n r}$

Explicit formula:

1st term $= a_1$

2nd $= a_1 r$

3rd $= a_1 r^2$

nth $= \boxed{a_1 r^{(n-1)} = a_n}$

Factorial: $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n$

On the calculator: MATH, →, →, →, 4

~~Geometric Mea~~

Arithmetic Means: terms between two given terms of an arithmetic sequence. When there is only one missing term, it is the average, or arithmetic mean.

Geometric means: terms between two given terms of a geometric ~~series~~ sequence. When only one term is missing, it is the mean proportional or geometric mean.

~~Sequence of part~~

Associated with every sequence a_n is another sequence, S_n, the sequence of partial sums.

$S_1 = a_1$

$S_2 = a_1 + a_2$

$S_n = a_1 + a_2 + \ldots + a_n$

S_n is the sum of the first n terms of the sequence a_n.

Series: an expression that ~~consists~~ consists of the terms of a sequence, alternating with the + sign. It is the ~~indicated sum of the terms of a sequence.~~ ~~A series is an~~ expression of the sum of the terms of a sequence (It is an expression, not a number).

Sigma/summination notation 20D

Upper limit → $$\sum_{n=y}^{x} 2n$$ "The sum of 2n from $n=y$ to $n=x$"

lower limit → n=y

index

Example: $$\sum_{n=1}^{5} 2n$$ $\{1,2,3,4,5\}$ = domain of summination

expanded form → $= 2(1)+2(2)+2(3)+2(4)+2(5)$

$= 2+4+6+8+10$

If the series is finite, the upper limit is an integer; if the series is infinite, it is ∞.

Sum of a...

Finite Arithmetic Series

$S_n = \frac{n}{2}(a_1+a_n)$

$S_n = \frac{n}{2}[2a_1+(n-1)d]$

Finite Geometric Series

$S_n = \frac{a_1 - r a_n}{1-r}$

$S_n = \frac{a_1(1-r^n)}{1-r}$

$S_n = \frac{a_1(r^n-1)}{r-1}$

Infinite Geometric Series

If $|r|<1$, the infinite geometric series

$a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \dots + a_1 r^{(n-1)} + \dots$

has the sum

$$S = \sum_{n=0}^{\infty} a_1 r^n = \frac{a_1}{1-r}$$

If $|r| \geq 1$, the series has no sum.